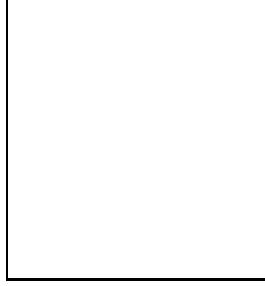# General soft terms from Supergravity including D-terms

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We derive general expressions for soft terms in supergravity where D-terms contribute significantly to the supersymmetry breaking. Such D-terms can produce large splitting between scalar and fermionic partners in the spectrum. By requiring that supersymmetry breaking sets the cosmological constant to zero, we the parameterize the soft terms when D-terms dominate over F-terms or are comparable to them. We present an application of our results to the split supersymmetry scenario and briefly address the issue of moduli stabilisation.

1 Introduction

A classical way of communicating supersymmetry breaking to the visible sector is through gravitational interactions. In supergravity[1], the hidden sector scalar potential is assumed to have a minimum, preferably generated dynamically, leading to a vacuum expectation value (vev) for at least one of the auxiliary fields. Tree level gravitational interactions then communicate this breaking to the visible sector generating soft terms in the global limit. General expressions for these soft terms can then be derived in terms of these auxiliary fields as has been pointed out long ago by [2]. By their very nature, such general expressions can be applied to study the soft terms in several classes of models such as supergravity lagrangians derived from superstring theories[3].

While the existing expressions have been extremely useful, they could be considered in a certain way as incomplete as they have been concentrating solely on the F type supersymmetry breaking terms. It is well known that there could be D type contributions too [4], that can arise for example in models based on anomalous $U(1)$ symmetries [5]. Furthermore, in effective

lagrangians from the Type II orientifolds with intersecting D-branes, one can expect such D-term contributions to be naturally present. There is also a second motivation. Recently, influenced by the multivacuum structure of string theory[6] as possible new view of cosmological constant problem, a new proposal has been put forward by the authors of Ref.[7]. Here, it is proposed that the fermionic superpartners stay close to the weak scale, whereas the scalar superpartners can be present at scales as high as 10^9 GeV. It would be very difficult to achieve this kind of splitting between superpartners in supergravity models with only F type supersymmetry breaking. Given these motivations, we present here the results obtained in [9] for the general expressions for soft terms in presence of non-zero D-term contributions and study a few applications for them. Particularly, we sketch a model where such large D-terms can be utilised in realising split supersymmetry and address the issue of moduli stabilisation, of particular relevance for any scenario of supersymmetry breaking.

2   General Expressions Including D-breaking

Let us now proceed to generalise the analysis in the literature by including abelian gauge groups $\prod_A U(1)_A$ and the corresponding D-type contributions to the SUSY breaking. The scalar potential now takes the form:

$$V = e^G(G^M\ G_M - 3) + \frac{1}{2}\sum_A g_A^2 D_A^2, \tag{1}$$

where the auxiliary F fields are given by $G_M = \frac{\partial G}{\partial z^M}$ and z represents the scalar part of a chiral superfield. The index M runs over all the chiral superfields present, matter as well as hidden sector and/or moduli fields. At the minimum, the hidden sector auxiliary fields attain a vev breaking supersymmetry spontaneously. The D-terms are given by

$$D_A = z^I X_I^A \frac{\partial K}{\partial z^I}\ +\ \xi_A = \bar{z}^I X_I^A \frac{\partial K}{\partial \bar{z}^I}\ +\ \xi_A\ ;\ \xi_A \equiv \eta_A^\alpha\ \partial_\alpha K, \tag{2}$$

where X_I^A represents the $U(1)_A$ charges of the field ϕ^I and ξ_A denotes the Fayet-Iliopoulos terms for the abelian $U(1)$ factors. Note that the equality between the first two terms is a straightforward consequence of the gauge invariance of the Kähler potential. We consider the Fayet-Iliopoulos terms to be moduli dependent and we will not explicitly discuss here the various possible mechanisms of moduli stabilisation. The conditions of the cancellation of the cosmological constant and the requirement of existence of a minimum gives

$$< e^G(G^M\ G_M - 3) + \frac{1}{2}\sum_A g_A^2 D_A^2 > = 0\ , \tag{3}$$

$$< e^G(G^M\ \nabla_K G_M + G_K) + \sum_A g_A^2 D_A(\partial_K D_A - \frac{1}{2}G_K D_A) >= 0\ , \tag{4}$$

where ∇ denotes the covariant derivative on the Kähler manifold. The scalar soft spectrum is defined as :

$$m_{I\bar{J}}^2 =\ < \partial_I \partial_{\bar{J}} V >\ =\ < \nabla_I \nabla_{\bar{J}} V >\ , \tag{5}$$

$$m_{IJ}^2 =\ < \partial_I \partial_J V >\ =\ < \nabla_I \nabla_J V >\ , \tag{6}$$

$$A_{IJK} =\ < \nabla_I \nabla_J \nabla_K V >\ . \tag{7}$$

We will further distinguish the visible sector (matter) fields from those of hidden sector fields T^α (and later on flavon fields), by requiring $\langle G^i \rangle = 0$, $\langle \Phi^i \rangle = 0$, with Φ representing the scalar part of a matter field. Using these we recover in the absence of D-term contributions the

standard form[8,3] for the soft scalar masses. Given the form of D-terms above (2), we have in the vacuum, after setting the matter fields *vevs* to zero

$$\langle \partial_j D_A \rangle = \langle \bar{v}_{\bar{\beta}} X^A_{\bar{\beta}} K_{\bar{j}\beta} + \eta^{\bar{\alpha}}_A K_{j\bar{\alpha}} \rangle = 0 \quad , \quad \langle \nabla_i \nabla_j D_A \rangle = 0 \ ,$$

$$\langle \partial_i \partial_{\bar{j}} D_A \rangle = K_{i\bar{j}} X^A_i + (\bar{v}^{\bar{l}} X^{\bar{l}}_A \partial_{\bar{l}} + \eta^{\bar{\alpha}}_A \partial_{\bar{\alpha}}) \, K_{i\bar{j}} \ , \ \langle \nabla_i \nabla_j \nabla_l D_A \rangle = 0 \tag{8}$$

The equations for the soft terms are now given by[9]:

$$m^2_{i\bar{j}} = m^2_{3/2} \left(G_{i\bar{j}} - R_{i\bar{j}\alpha\bar{\beta}} G^\alpha G^{\bar{\beta}} \right) + \sum_A g^2_A D_A \left(X^A_i + \bar{v}_{\bar{l}} X^A_{\bar{l}} \partial_{\bar{l}} + \eta^{\bar{\alpha}}_A \partial_{\bar{\alpha}} - \frac{1}{2} D_A \right) G_{i\bar{j}} \ , \tag{9}$$

$$m^2_{ij} = m^2_{3/2} \left(2\nabla_i G_j + \ G^\alpha \nabla_i \nabla_j G_\alpha \right) - \frac{1}{2} \sum_A g^2_A D^2_A (\nabla_i G_j + \frac{1}{2} g^2_A \partial_i \partial_j f_A) \ , \tag{10}$$

$$A_{ijk} = m^2_{3/2} \left(3\nabla_i \nabla_j G_k + G^\alpha \nabla_i \nabla_j \nabla_k G_\alpha \right) - \frac{1}{2} \nabla_i \nabla_j G_k \sum_A g^2_A D^2_A \ , \tag{11}$$

where we have identified the gravitino mass $< e^G >= m^2_{3/2}$ and f_A is the gauge kinetic function. While these expressions are given for the tree level potential, higher order corrections can play a significant role, depending on the specifics of the model of supersymmetry breaking. In models with small tree-level contributions, the dominant set of corrections are of anomaly mediated type[10] which are proportional to the gravitino mass $m_{3/2}$. A detailed analysis including various parameterisations will be presented in[9]. The μ term and the tree level gaugino mass terms are then given by

$$\mu_{ij} = m_{3/2} \, \nabla_i G_j \quad , \quad M^A_{1/2} = \frac{1}{2} (Ref_A)^{-1} m_{3/2} f_{A\alpha} G^\alpha \ . \tag{12}$$

It is clear from the above analysis that in the F-limit where D term contributions are absent, the soft terms all typically of the same order of magnitude without large hierarchies within themselves. These expressions have been used to parameterise soft terms from superstring theories as well as supergravity[3]. Of course, if the gravitino mass is very larg $m_{3/2} >> TeV$, possible higher derivative operators can change the pattern displayed above completely. A consistent supergravity analysis in such a case, however, becomes considerably more involved.

2.1 Implications of large D-terms on the soft parameters

Let us now systematically see the impact of the D-terms on each of the soft parameters. As has been noted earlier, as long as they are of the $\mathcal{O}(m^2_{3/2})$, they do not have strong impact. Let us now consider the limit $m^2_{3/2} \lesssim D_A \lesssim m_{3/2} M_P$.

(i). *Sfermion Mass Terms*: The most dominant contribution to the sfermion masses from the D-terms are the ones which are linear in D which for $m_{3/2} \sim$ TeV push the scalar masses to intermediate energy scale. Note that these terms depend on the charges of the fields under the additional $U(1)$ gauge group, thus putting a constraint that these charges to be of definite sign. If all the three generations of the sfermions have the same charges under the $U(1)$ groups, this term would also be universal.

(ii). *Higgs mass terms and the $B\mu$*: The Higgs masses follow almost the same requirements as the soft masses. Usually, their charges are linked with the Giudice-Masiero mechanism[11]. The $B\mu$ term is however special. Unlike the Higgs mass terms, it does not receive large contributions from D-terms, whose contributions can be utmost of $\mathcal{O}(m^2_{3/2})$. If the splitting between the Higgs masses and the $B\mu$ is too large, it could lead to unphysical regions in $\tan\beta$. To remedy this, alternative schemes have to be devised, an explicit example being discussed in the next subsection.

(iii). *A-terms*: Even if the D-terms are large, the A-terms are typically proportional to $\mathcal{O}(m_{3/2})$. No large enhancement is present. This is expected as A-terms break R-symmetries.

2.2 A model for Split supersymmetry

The requirement of Split supersymmetry type soft spectra are as follows :

(i) Scalar soft terms : $m_{\tilde{f}}^2 \sim \mathcal{O}(10^9 - 10^{15})$ GeV, $(\tilde{f} = Q, u^c, d^c, L, e^c)$

(ii). Higgs mass parameters $m_{H_1}^2 \sim m_{H_2}^2 \sim B_\mu \sim \mathcal{O}(10^9 - 10^{15})$ GeV, with one of them fine tuned to be around the electroweak scale.

(iii). The gaugino masses and the μ term are around the weak scale.

From the discussion in the previous section, it was obvious that it is just not sufficient to choose the $U(1)$ charges of the sfermions to be positive to realise the split spectrum^c. Since B_μ term does not have large D-term contributions, we need to disentangle the μ and the B_μ term by introducing (at least) one new field X and allowing a term of the type XH_1H_2 in the superpotential. If the auxiliary field $\langle F_X \rangle \sim \langle D \rangle$, whereas $\langle X \rangle << m_{3/2}$, then the tuning of Higgs parameters is technically possible. The minimal field content realising this is as follows. The model contains an additional $U(1)$ group, with two additional fields X and ϕ with charges $+2$ and -1. The ϕ field can act as a flavon field attaining a large vev close to the fundamental scale. The superpotential and the relevant terms in the Kahler potential, obtained by expanding in powers of the matter fields the full supergravity , are

$$W = W_0 + W_{SSM} + \lambda_1 X H_1 H_2 + \lambda_2 X \phi^2 + \cdots ,$$
$$K = K_0 + \sum_{i\bar{j}} Z_{i\bar{j}} \phi^i \bar{\phi}^{\bar{j}} + Z'(\phi^\dagger)^2 H_1 H_2 + \cdots . \tag{13}$$

In (13), W_0 is a holomorphic function of moduli fields , K_0 is the Kahler potential for moduli, $Z_{i\bar{j}}$ and Z' are generically also moduli dependent and the dots denote higher order terms in an expansion in matter fields. The main features of the model are already captured by performing a vacuum analysis at the global supersymmetry level. In this case, the scalar potential is given by

$$V = \lambda_2^2(|\phi|^4 + 4|X|^2|\phi|^2) + \frac{1}{2}g^2 \left(2|X|^2 - |\phi|^2 + \xi\right)^2 + \cdots , \tag{14}$$

For $\xi > 0$, the stable extremum of the above and the auxiliary fields are given by:

$$\langle\phi\rangle = \frac{g^2}{2\lambda_2^2 + g^2}\xi , \ \langle X\rangle = 0 , \ \langle F_\phi\rangle = 0 , \ \langle F_X\rangle = \frac{\lambda_2 g^2}{2\lambda_2^2 + g^2}\xi , \ \langle D\rangle = \frac{2\lambda_2^2}{2\lambda_2^2 + g^2}\xi . \tag{15}$$

From the above it is clear that $F_X \sim g^2 D$ and moreover are of order of the FI term ξ. This is sufficient to enable the B term to receive large contributions through the term $G^X \nabla_{H_1} \nabla_{H_2} G_X$ in the eq.(10). As long as ξ is close to an intermediate scale $m_{3/2}^2 << \xi \leq m_{3/2}M_P$, this model seems to replicate the hierarchical split spectrum, if one fixes the gravitino mass at 1 TeV. However, in typical string models, the FI term is of the $\mathcal{O}(M_{Pl}^2/16\pi^2)$, which would give a too large contribution to the vacuum energy. The correct order of magnitude could be achieved by incorporating the above model into a higher dimensional theory. For illustration lets us consider a 5D theory compactified over S^1/Z_2. The Standard Model and the X, ϕ fields live on a 3D brane, whereas the gauge fields of the $U(1)$ are allowed to propagate in the bulk. We will use Scherk-Schwarz mechanism to break supersymmetry.

The various scales in the problem are $R = tM_5^{-1}$, $RM_5^3 = M_P^2$, where $t \equiv Re\ T$, the modulus field. After canonically normalizing the various fields by $\hat{\phi}_i = \sqrt{t/3}\ \phi_i$ and at the global supersymmetry level, the potential retains the form (14) with $\xi \sim M_5^2 = M_P^2/t$. The four dimensional gauge coupling is given by $g^2 = 1/t = 1/(RM_5)$, whereas the gravitino mass is given by $m_{3/2} = \omega/R$, where ω is a number of order one. The D-term contribution to the vacuum energy is then of the form $\langle V_D \rangle \sim g^2 M_5^4 \sim m_{3/2}^2 M_P^2$, in the right order as required by the

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^{c}see also Ref.[12]

cancellation of the vacuum energy and realisation of the split spectrum. If the no-scale structure is broken by the dynamics, the gauginos attain their masses through anomaly mediation and thus we choose the gravitino mass to be of the order of 100 TeV. In the opposite case, new sources of gaugino masses have to be invoked, like Dirac-type masses or higher dimensional operators if the gravitino is much heavier. The μ term can be generated by Giudice-Masiero mechanism and is $\mu \sim (v/M_5)^2 m_{3/2}$. So, this model replicates the spectrum of the split supersymmetry at the weak scale using large D-terms of the intermediate scale and a 100 TeV massive gravitino.

2.3 Moduli stabilisation problem

As transparent in (2), the FI terms are field (moduli) dependent. If no additional dynamics is present, the moduli fields will always exbihit a runaway behaviour and the FI terms disappear. We resume here the issue of moduli stabilisation with realisation of large D-term contributions to soft terms discussed in [9] in a context similar to, but having some new features compared the one discussed some time ago in [5]. We would like to stress that the analysis performed in [9] and summarized here is also relevant for the issue of the uplift of the energy density in the context of KKLT type moduli stabilisation [13]. The gauge group consists of the Standard Model supplemented by a confining hidden sector group and an anomalous $U(1)_X$. For simplicity we discuss the case of an supersymetric $SU(2)$ gauge group with one quark flavor Q^a and anti-quark $\tilde{Q}^a$ where $a = 1, 2$ is an index in the fundamental representation of the $SU(2)$ gauge group. The hidden sector consists of a stack of two magnetised D9 branes in the type I string with kinetic function $f = S + kT$, where S is the dilaton (super)field, T a volume (Kahler) modulus and k is an positive or negative integer determined by the magnetic fluxes in two compact torii. The dynamical scale of the hidden sector gauge group and the effective superpotential are

$$\Lambda = M_P e^{-8\pi^2(S+kT)/5} \ , \ W = W_0 + \frac{\Lambda^5}{M} + \lambda\varphi M \ . \tag{16}$$

In order to stabilise the modulus S we invoke the three-form NS-NS and RR fluxes. The low energy dynamics is described by $M = Q^a \tilde{Q}^a$, the composite "meson" field. W_0 depends on the modulus S and eventually other (complex structure) moduli of the theory and stabilise them $S = S_0$ by giving them a very large mass. If the other relevant mass scales, the FI term and the dynamical scale Λ have much lower values, we can safely integrate out these fields, by keeping the T modulus in the low energy dynamics. Minimisation with respect to T stabilises also the Kahler modulus. Due to the anomalous nature of the $U(1)_X$, there are mixed anomalies with the hidden sector gauge group which translate into a chiral nature of the quark and anti-quark field, such that the sum of their charges, equal to the M meson charge, is different from zero and, in our example, equal to $+1$. φ is a field of charge -1 which originally participate in the Yukawa coupling $\lambda\varphi Q^a \tilde{Q}^a$, which plays the role of meson mass after the spontaneous symmetry breaking of the $U(1)_X$. Along the $SU(2)$ flat directions, the D-term scalar potential is

$$V_D = \frac{g_X^2}{2} \left[(M^\dagger M)^{1/2} - |\varphi|^2 + k\mu^2 \right]^2 \ , \tag{17}$$

where μ is a mass scale determined by the T-modulus vev. The new feature of (17) is that k and consequently the FI term can have both signs, whereas in the effective heterotic string framework worked out in [5], the FI term had only one possible sign. In the limit $\Lambda << \mu$, the vacuum structure and the pattern of supersymmetry breaking in the two cases of k positive and negative are vastly different :

i) $k > 0$. In this case the vacuum can be determined as in [5]. We find, to the lowest order in the parameter $\epsilon \simeq (\Lambda/k^{1/2}\mu)^{5/2}$, a hierarchically small scale of supersymmetry breaking

$$\langle|\varphi|^2\rangle = k\mu^2 \ , \ \langle M \rangle = \lambda^{-1/2}\Lambda^2(\Lambda/k^{1/2}\mu)^{1/2} \ ,$$

$$\langle D_X \rangle = -\frac{\lambda \Lambda^5}{(k^{1/2}\mu)^3} \ , \ \langle F_\varphi \rangle = \Lambda^2 (\frac{\lambda \Lambda}{k^{1/2}\mu})^{1/2} \ , \ \langle F^{\bar{M}} \rangle = K^{M\bar{M}} \partial_M W = -\frac{\Lambda^5}{k\mu^2 M_P^2} \ . \quad (18)$$

ii) $k < 0$. In this case we find, to the lowest order in the parameter $\epsilon' \sim (\Lambda^2/|k|\mu^2)^5$, a large scale of supersymmetry breaking (for the complete expressions, see [9])

$$\langle |\varphi| \rangle \sim \frac{\Lambda^5}{k^2\mu^4} \ , \ \langle M \rangle \sim |k|\mu^2 \ , \ \langle D_X \rangle \sim k\mu^2 \ , \ \langle F_\varphi \rangle \sim k\mu^2 \ , \ \langle F^{\bar{M}} \rangle \sim -\frac{\Lambda^5}{k\mu^2 M_P^2} \ . \quad (19)$$

Interestingly enough, this second case generate a large scale for supersymmetry breaking with large D-term contributions.

2.4 Acknowledgments

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